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A Probabilistic Method to Generate Artificial Earthquake Ground Motions

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ABSTRACT

Generating artificial earthquake ground motions has been of interest for structural engineers for a long time. A method has been proposed to generate earthquake ground motion time histories. With this method, the problem of shortage of recorded ground motions can be overcome and it will be helpful in performance based earthquake engineering. The shortcomings of traditional methods such as scaling and spectrum matching are overcome in this method as it reproduces the real characteristics of the earthquake. The method is based upon a stochastic ground motion model, which has separable temporal and spectral non-stationarities. The ground motion model uses six parameters to represent the time duration, amplitude and the shape of the decaying end (temporal non-stationarity) and three parameters to represent the predominant frequency and bandwidth (spectral non-stationarity) of the process. A database of recorded earthquake ground motions is created. In the next step, the nine parameters required to depict a particular ground motion are determined for all the ground motions in the database. Probability distributions are created for the parameters of all the earthquakes in the database. Now, the parameters required by the stochastic ground motion model to simulate ground motions are obtained from the distributions. Monte Carlo Simulations can be used to generate a huge number of ground motions. As an example, 100 arbitrarily chosen recorded earthquake ground motions are collected and the method is demonstrated. The method can be used to generate ground motions of specific characteristics by using already recorded ground motions of similar characteristics.

Keywords: Artificial Earthquake, Monte Carlo Simulation, Probability Distribution, Stochastic Process

3. INTRODUCTION

Earthquake ground motions are non-stationary in both time and frequency domains. Temporal non-stationarity refers to the variation in the intensity of the ground motion in time. Spectral non-stationarity refers to the variation in the frequency content of the motion in time. Although temporal non-stationarity can be easily modelled by multiplying a stationary process by a time function, spectral non-stationarity is not so easy to model. However, both effects are important, particularly in the non-linear response analysis. The modelling, analysis and simulation of ground motion signals is of crucial importance in studying and improving the behaviour of structures under earthquake excitation.

The modelling, analysis and simulation of ground motion signals is of crucial importance in studying and improving the behaviour of structures under earthquake excitation and has thus attracted significant attention during the past several years. The growing interest in performance-based earthquake engineering (PBEE) in recent years has further increased the need for stochastic modelling of ground motions. The PBEE analysis typically considers the entire spectrum of structural response, from linear to grossly non-linear and even collapse. For such an analysis, realistic characterization of the ground motion is essential. In the current PBEE practice, usually recorded ground motions are employed, which are then scaled to various levels of intensity. This approach suffers from scarcity of recorded ground motions for specified earthquake characteristics. Stochastic ground motion models provide an alternative for use in PBEE in lieu of or in conjunction with recorded ground motions.

There are two types of stochastic ground motion models: models that describe the random occurrence of fault ruptures at the source and propagation of the resulting seismic waves through the ground medium (source based models) and models that describe the ground motion for a specific site by fitting to a recorded motion with known earthquake and site characteristics (site based models). A review of source based models is presented by Zerva (Zerva, 1988). By using a site based stochastic model, one is able to generate artificial ground motions, which have statistical characteristics similar to those of the target ground motion. A large number of site based models have been proposed in the past. A review is presented by Shinozuka and Deodatis (Shinozuka, 1988) and more recently by Conte (Conte, 1997).

Kiureghian (Kiureghian, 1989) proposed an evolutionary random process model for describing the earthquake ground motion. The model is composed of individually modulated component stationary processes, each component representing the energy in the process in a narrow band of frequencies. The model accounts for both temporal and spectral non-stationarity of the motion. A probabilistic ground motion model was proposed by Papadimitriou (1990) which is capable of capturing, with at most nine parameters, all those features of the ground acceleration history which have an important influence on the dynamic response of linear and non-linear structures, including the amplitude and frequency content non-stationarities of the shaking. The model is based on bayesian probabilistic framework. A fully non-stationary stochastic model for strong earthquake ground motion was developed by Rezaeian and Kiureghian (2008). The model employs filtering of a discretized white-noise process. Non-stationarity is achieved by modulating the intensity and varying the filter properties in time. The formulation has the important advantage of separating the temporal and spectral non-stationary characteristics of the process, thereby allowing flexibility and ease in modelling and parameter estimation.

There is a need to develop a probabilistic model to generate ground motions of required characteristics. A model based on actual recorded ground motions has been developed and presented here.

STOCHASTIC GROUND MOTION MODEL

The stochastic ground motion model used in the present study considers both the temporal and spectral non-stationarities. The selected model has the following advantages: 1. The model has a small number of parameters, which control the temporal and spectral non-stationary characteristics of the simulated ground motion and can be easily identified by matching with similar characteristics of the target accelerogram. 2. The temporal and spectral non-stationary characteristics are completely separable, facilitating identification and interpretation of the parameters. 3. There is no need for sophisticated processing of the target accelerogram, such as the Fourier analysis or estimation of evolutionary power spectral density. 4. The filter model provides physical insight and its parameters can be related to the characteristics of the earthquake and site considered. 5. Simulation of sample functions is simple and requires little more than generation of standard normal random variables.

The ground acceleration, at any time *t* is given by,

, (1)

where is the modulating function at time *t.* is a standard random normal variable. A modified version of the Housner and Jennings model, Housner (1964) as stated below is used as the modulating function.

,

,

,

, (2)

This model has six parameters , , , , and which obey the conditions *, ,* and . denotes the start time of the process, and denote the start and end times of the strong-motion phase with root mean square (RMS) , and and are parameters that shape the decaying end of the modulating function. In Housner and Jennings model is taken as 1.
k= int where is the time steps taken for discretizing the model,

, , (3)

Any damped single- or multi degree-of-freedom linear system that has differentiable response can be selected as the filter, here

,

, otherwise (4)

which represents the pseudo-acceleration response of a single-degree-of-freedom linear oscillator subjected to a unit impulse, in which denotes the time of the pulse. is the set of parameters of the filter with denoting the natural frequency and denoting the damping ratio, both dependent on the time of application of the pulse. , influence the predominant frequency of the resulting process, whereas influence its bandwidth. The predominant frequency of an earthquake ground motion tends to decay with time. Therefore,

(5)

in which is the total duration of the ground motion, is the filter frequency at time and is the frequency at time . For a typical ground motion, . Thus, the two parameters and describe the time-varying frequency content of the ground motion.

Parameter Identification

As shown above, the temporal and spectral characteristics of the model are completely separable. Specifically, the modulating function describes the evolving RMS of the process, whereas the filter IRF controls the evolving frequency content of the process. This means that the parameters of the modulating function and of the filter can be independently identified by matching to corresponding statistical characteristics of a target accelerogram.

Identification of parameters in the modulating function

Let **α**=*(, , , , ,)* denote the parameters of the modulating function, so that = . For a target accelerogram, *a(t)*, we determine **α** by matching the expected cumulative energy of the process, , with the cumulative energy in the accelerogram, , over the duration of the ground motion, . This is done by minimizing the integrated squared difference between the two cumulative energy terms, i.e.,

(6)

Identification of parameters in the filter function

The parameters and defining the time-varying frequency of the filter in Eq (5) and parameters defining its damping ratio control the predominant frequency and bandwidth of the process. Since these parameters have interacting influences, we first determine and while keeping the filter damping a constant, . For a given , the parameters and are identified by matching the cumulative expected number of zero-level up-crossings of the process, i.e. , with the cumulative count of zero-level up-crossings in the target accelerogram for all *t*, . This is accomplished by minimizing the mean-square error,

(7)

where , is the mean number of times per unit time that the process crosses the level zero from below is used. Since the scaling of a process does not affect its zero-level crossings, for the process in Eq (1) is identical to that for the process,

, (8)

It is known that for such a process,

(9)

where , and are the standard deviations and cross-correlation coefficient of *y(t)* and its time derivative, at time *t*. For the process in Eq (9), these are given by,

, (10)

, (11)

, (12)

where . Using Eq (3) and , it's shown that,

, , (13)

where is an adjustment factor as described below. is an implicit function of the filter characteristics and and therefore, , and . The same is true for .

When a continuous-parameter stochastic process is represented as a sequence of discrete-time points of equal intervals , the process effectively loses its content beyond a frequency approximately equal to rad/s. This truncation of high-frequency components results in undercounting of level crossings. The undercount per unit time, denoted as *r*, is a function of as well as the frequency characteristics of the process. So, *r* is a function of , and . Approximate expressions for *r* are,

when, (14)

when, (15)

Since digitally recorded accelerograms are available only in the discretized form, the count underestimates the true number of crossings of the target accelerogram by the factor per unit time. Hence, to account for this effect, we must multiply the rate of counted up-crossings by the factor 1/. However, depends on the predominant frequency and bandwidth of the accelerogram. So, it is more convenient to adjust the theoretical mean up-crossing rate (the first term inside the square brackets in Eq (7)) by multiplying it by the factor. In order to solve Eq (7), the filter damping ratio, which controls the bandwidth of the process is selected. Corresponding and are calculated. The value of which best fits the target is chosen.

STOCHASTIC SIMULATION OF GROUND MOTIONS

A probabilistic method to generate an ensemble of artificial earthquake ground motions based on the stochastic ground motion model described earlier is proposed. A database of recorded earthquake ground motions is created. In the next step, the nine parameters required to depict a particular ground motion is found out for all the ground motions in the database. Probability distributions are created for the parameters of all the earthquakes in the database. Now, the parameters required by the stochastic ground motion model to simulate ground motions are obtained from the distributions. Monte Carlo simulations are used to generate an ensemble of ground motions.

A database of recorded earthquake accelerograms is created. The earthquakes are selected arbitrarily. Earthquakes with intensity varying from moderate to high that have occurred throughout the world in the past century are chosen. The chosen earthquakes have occurred on different site conditions and have different characteristics. Earthquake data are collected from reliable sources. Details of the earthquakes in the database is given in Jacob (2010).

Determination of Parameters

The nine parameters of the stochastic ground motion model are found out for each of the earthquakes in the database. The two additional parameters that are required are the time duration T_n and the time steps Δt. T_n is known from the time history and Δt is taken as 0.02 sec for all the earthquakes. As the temporal and spectral non-stationarities are separable in the considered stochastic ground motion model, the parameters of the modulating function and the parameters of the filter are found out separately. The results are shown for one accelerogram. The accelerogram chosen is the October 18th, 1989 Loma Prieta earthquake's 90 component recorded at Los Gatos Presentation Centre.

Parameters in the modulating function

The six parameters required for the modulating function can be obtained by solving Eq. (6) by using an optimization technique. The minimum of the unconstrained multivariable function in Eq. (6) is obtained by using Nelder-Mead simplex algorithm (Wright, 1998). A MATLAB code is developed for this purpose. Figure 1 compares the two energy terms. It is seen that the fit is good at all the time. points. The error is minimised. The parameters obtained after optimization for , , , , and are 0.072932 sec, 8.0154 sec, 12.88 sec, 0.16308g, 0.80585 sec^{-1} and 0.44846, respectively.

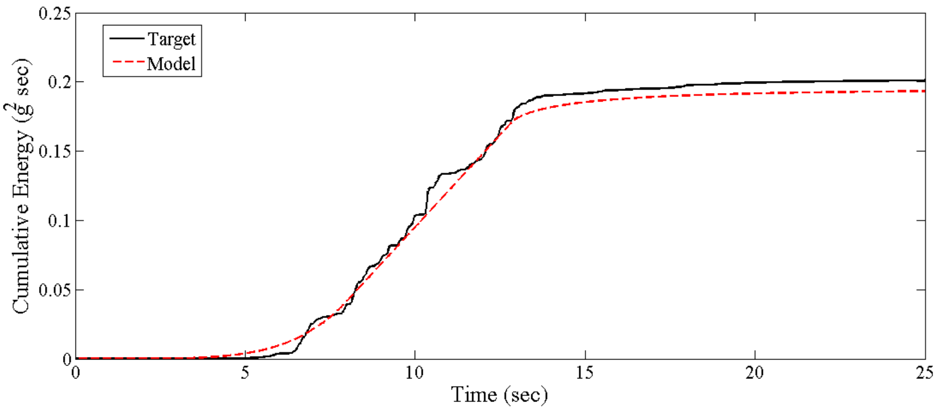

Figure 1. Cumulative energies in the target accelerogram and the fitted model

Parameters in the filter function

Using the same method of optimization mentioned before, the parameters in the filter are obtained by solving Eq (7). To solve Eq (7), first a value of ζ_f is assumed. ζ_f is considered to be constant for the entire duration of the earthquake. After obtaining the values of ω_0 and ω_n, separate optimization is done by minimizing the difference between the cumulative count of negative maxima and positive minima of the target and the model. The values of ω_0 and ω_n are kept unchanged as it is found that there is no big variation. Now, the exact value of ζ_f is known. The values of ω_0 and ω_n are found corresponding to the final ζ_f. For the considered accelerogram it is found to be 0.8. Shown in Figure 2 is the cumulative count of negative maxima and positive minima as a function of time for the Loma Prieta, 1989 record as well as the estimated values of the same quantity of the model with damping ration ζ_f = 0:8. The slope of these lines should be considered as the instantaneous measure of bandwidth. The values of ζ_f, ω_0 and ω_n are found to be 0.8, 30.297 rad/sec and 10.075 rad/sec respectively. Shown in Figure 3 is the cumulative number of zero-level up-crossings as a function of time for the Loma Prieta, 1989 record as well as the estimated values of the same quantity of the model. It is seen that the fit is good at all the time points.

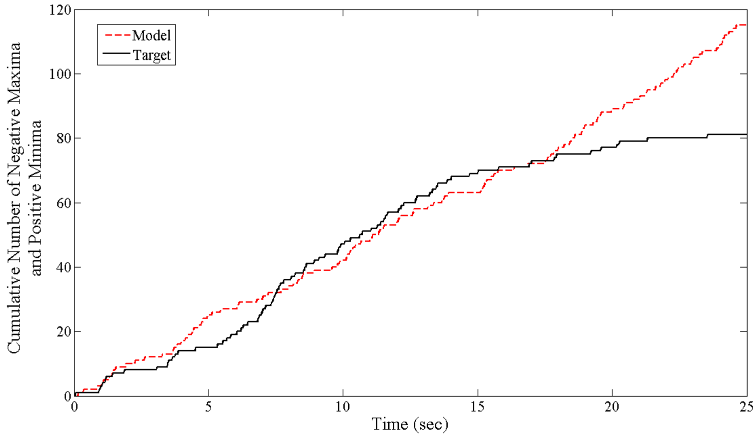

Figure 2. Cumulative count of negative maxima and positive minima

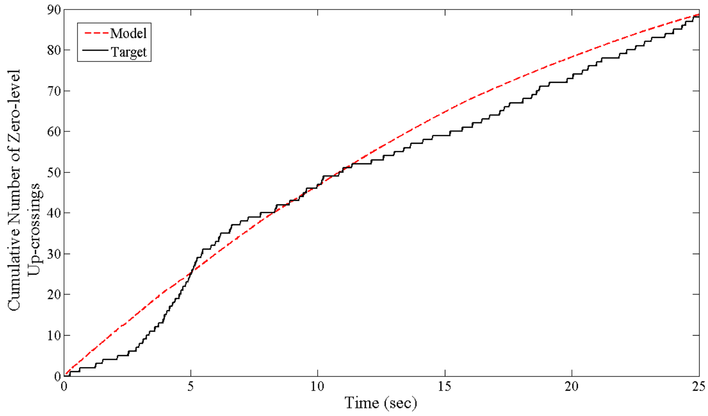

Figure 3. Cumulative number of zero-level up-crossings in the target accelerogram and model

Simulation of a Target Accelerogram

After finding out all the parameters, an accelerogram can be simulated by using the process described earlier. Simulated accelerogram has similar characteristics but it will not be an exact replica of the target accelerogram. The target accelerogram and a simulation are shown in the Figure 4. To get more accurate simulations, the damping ratio of the filter ζ_f should be considered as varying at various intervals of time. For simplicity, the damping ratio of the filter ζ_f is considered as constant throughout the entire duration of the earthquake. Since we had described the ground acceleration as a filtered white noise process which has a non-zero spectral density at zero frequency, the integral of the process (the ground velocity or displacement) has infinite spectral density at zero frequency. Because of this property, the variances of the velocity and displacement processes keep on increasing even after the acceleration has vanished. This is contrary to (baseline- corrected) accelerograms, which have zero residual velocity and displacement at the end of the record. To overcome this problem, it is necessary to adjust the low frequency content of the stochastic model using a high-pass filter. This is not done in the present work.

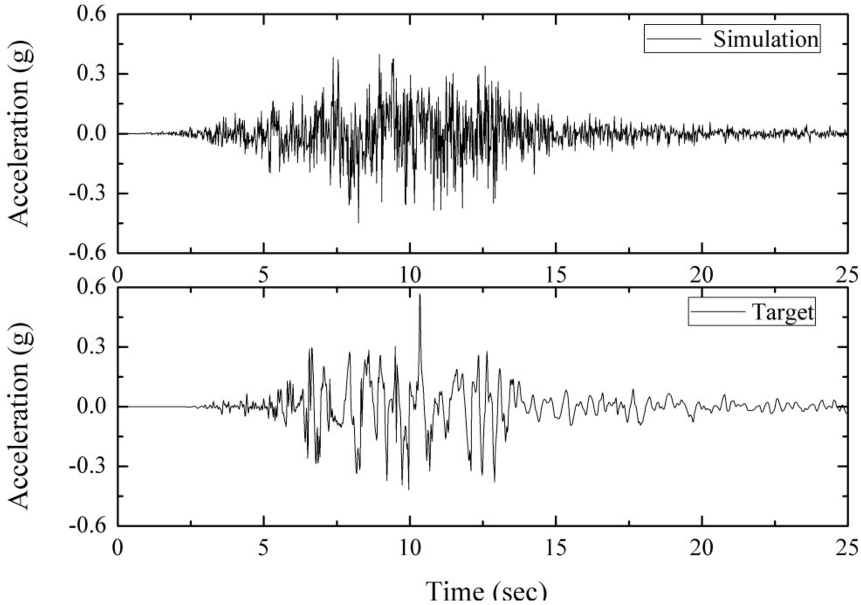

Figure 4. Target accelerogram and a simulation using the fitted model

Distribution of Parameters

After identifying the model parameter values by fitting to each recorded ground motion in the database, a probability distribution is assigned to the sample of values of each parameter. Now, there are 100 sets of parameters representing the 100 earthquakes in the database. The time steps Δt is kept constant for all the 100 earthquakes. Distribution models are assigned to each of the 10 parameters. It is found that the data of all the 10 parameters are effectively fitted by β distribution. Figure 5 shows the normalized frequency diagrams of the fitted model parameters for the entire dataset with the fitted probability density functions (PDFs) superimposed.

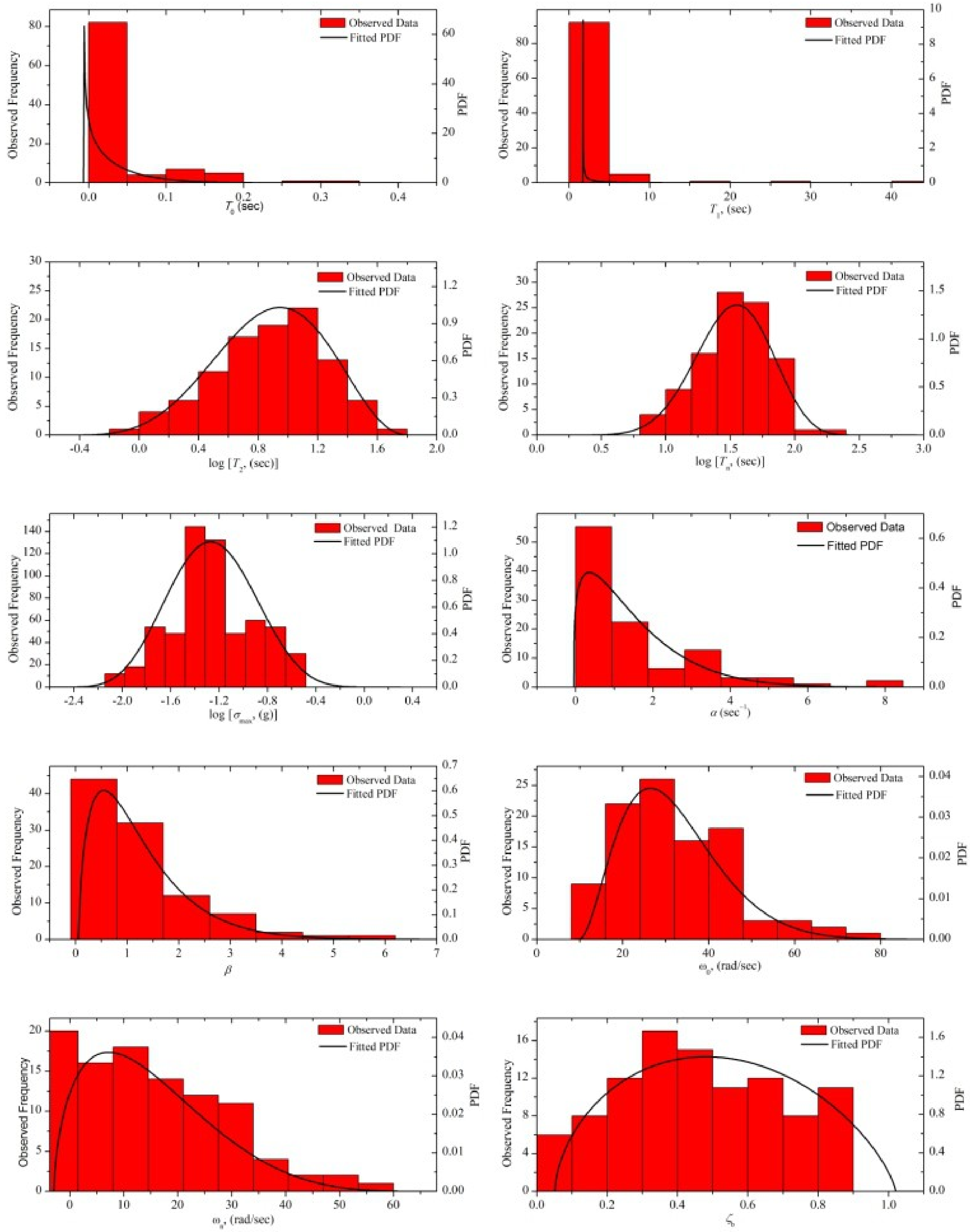

Figure 5: PDF of parameters superimposed on observed normalized frequency diagrams

Generation of an Ensemble of Ground Motions

A cluster of earthquake ground motions can be produced. This can be done by randomly selecting the parameters of the stochastic ground motion model from the distributions of the particular parameter. From this pool of samples, sets of parameters which satisfy the conditions specified earlier can be chosen. Using these sets of parameters, artificial ground motions can be generated. This cluster of earthquakes represents a completely random choice of ground motions.

When ground motions of any specific characteristic is required, a small database containing earthquakes of similar characteristics that have occurred already should be collected and the model proposes can be applied to generate an ensemble of earthquake ground motions.

4. CONCLUSIONS

The probabilistic model developed based on arbitrary recorded earthquakes can be used to study the response of structures under random earthquake excitations. When studies using specific characteristics of earthquakes such as site conditions, fault type, etc. are to be done, the model can be easily applied by using a suitable database. Since the model is based on actual recorded earthquakes, ground motions corresponding to actual characteristics of the earthquake can be reproduced.

An extensive database with earthquakes with specific characteristics like fault type, site characteristics, etc. can be developed. The model developed can be used to generate an earthquake of desired qualities. The predictive equations used to predict the earthquake characteristics for a particular site can be combined with the ground motion model to produce a set of earthquakes pertaining to that site. By doing this, it will be possible to generate a ground motion for a particular site with an associated probability of occurrence of which will be very useful for design engineers.

5. ACKNOWLEDGMENTS

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